A Stochastic Two Settlement Equilibrium Model for Electricity Markets with Wind Generation

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***Abstract*—Incentives to encourage the uptake of renewable energy generation have fostered wind energy in many power systems. These incentives usually take the form of market instruments (e.g. feed-in tariff or premium) that are not directly amenable to optimization representations of the market. In this paper, we propose an equilibrium model of the short term market to address the impact of wind operation under different structural assumptions. The model is formulated for several price taking, risk averse firms in competition. It accounts for wind generation uncertainty and embeds a representation of the day ahead and balancing mechanisms. The consumer is modeled by a linear inverse demand function. We focus on feed-in premium as the incentive to wind as this is the instrument most favored today in European discussions.**

The model is formulated as a stochastic equilibrium problem where the Karush-Kuhn-Tucker (KKT) conditions from the optimization problem of each firm are simultaneously solved together with market clearing conditions on energy, capacity for reserve and energy for reserve. The problem for each firm consists of a two-stage stochastic optimization problem with a recourse function based on the Conditional Value at Risk, ($CVaR_\theta$), as a risk measure. Due to price taking assumptions the model is a single stage complementarity problem; it is implemented and solved using the software GAMS. An example based on a stylized simplification of the Spanish power market and motivated by the impact of wind penetration on the revenue of conventional plants is used to illustrate the proposed approach.

***Index Terms*—CVaR, equilibrium, market, stochastic programming, Feed-in Premium.**

NOTATION

A. Indices and sets

- f, h, F Indices and set for firms, f, $h \in F$.
- G Set of all the generators.
- G_f Set of generators belonging to firm f.
- g Index for dispatchable generators, $g \in G$.
- l Index for wind generators, $l \in G$.
- k, Ω Index and set for scenarios, $k \in \Omega$.

B. Parameters

- $\overline{A}$ Upper bound for the requirement of committed upward reserve. In the Spanish System $\overline{A} = 110\%$.
- $\underline{A}$ Lower bound for the requirement of committed upward reserve. In the Spanish System $\underline{A} = 90\%$.
- B Lower rate for committed downward reserve respect to the committed upward reserve. In the Spanish System $B = 40\%$.
- $\overline{R}$ Upward ramping slope respect to the generation capacity, usually $\overline{R} \in [0.02, 0.53]$ per hour.
- $\underline{R}$ Downward ramping slope respect to the generation capacity, usually $\underline{R} \in [0.02, 0.53]$ per hour.
- cg_g Linear coefficient of generation cost for dispatchable generator g, [€/MWh].
- m_y Balancing reserve factor for wind turbines.
- m_x Balancing reserve factor for dispatchable generators.
- Pr^k Probability of scenario k.
- $\overline{X}_g$ Maximum generation capacity of dispatchable generator g, [MW].
- $\underline{X}_g$ Must-run capacity of dispatchable generator g, [MW]. In the case study $\underline{X}_g = 0$.
- $\overline{Y}_l^k$ Maximum generation capacity available for wind turbine l in scenario k, [MW].
- $\underline{Y}_l^k$ Must-run capacity of wind turbine l in scenario k, [MW]. In the case study $\underline{Y}_l^k = 0$.
- $\overline{Y}_l^{max}$ Installed capacity of wind turbine l, [MW].
- α_0 Slope of the inverse demand function [€/(MWh)^2].
- θ_f Confidence level for firm f CVaR, $\theta_f \in (0, 1)$.
- λ_f Level of risk aversion, $\lambda_f \in [0, 1]$, $\lambda_f = 0$ risk neutral.
- ρ_0 Independent coefficient of the inverse demand function, [€/MWh].
- ρ^+ Premium associated with the scheduled wind generation, [€/MWh].

C. Variables

Primal variables

- d_f Energy sales of firm f in day ahead, [MWh].
- $Q_f^{k,-}$ Auxiliary variable for $CVaR_\theta$ calculation.
- ru_g Committed upward reserve from dispatchable generator g, [MW].
- rd_g Committed downward reserve from dispatchable generator g, [MW].
- s_g^k Energy deployed from upward reserve of dispatchable generator g at scenario k, [MWh].
- u_g^k Energy deployed from downward reserve of dispatchable generator g at scenario k, [MWh].
- v_l^k Wind energy in excess over the scheduled generation for wind turbine l at scenario k, [MWh].
- x_g Scheduled generation of dispatchable unit g, [MWh].

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y_l Scheduled generation of wind turbine l, [MWh].
z_l^k Energy bought by wind turbine l at scenario k, [MWh].
ζ_f Auxiliary variable for $CVaR_\theta$ calculation.

Dual variables

$\overline{\gamma}$ Upper bound for committed upward reserve.
$\underline{\gamma}$ Upper bound for committed downward reserve.
$\overline{\delta}_g$ Upper bound for upward reserve that generator g can supply. Ramping constraint.
$\underline{\delta}_g$ Upper bound for downward reserve that generator g can supply. Ramping constraint.
$\overline{\eta}_g^k$ Upper bound for the energy from the committed upward reserve, dispatch. generator g, scenario k.
$\underline{\eta}_g^k$ Upper bound for the energy from the committed downward reserve, dispatch. generator g, scenario k.
$\overline{\iota}_l^k$ Upper bound for purchases of wind turbine l at scenario k.
$\underline{\iota}_l^k$ Lower bound for purchases of wind turbine l at scenario k.
$\overline{\kappa}$ Lower bound for committed upward reserve [€/MWh].
$\underline{\kappa}$ Lower bound for committed downward reserve [€/MWh].
ν_f Power balance of firm f.
ξ_l^k Upper bound for sales in excess of wind turbine l at scenario k.
σ_f^k $CVaR_\theta$ constraint.
τ_l^k Constraint that relates the available wind in excess for wind turbine l and it use at scenario k.
$\overline{\phi}_g$ Upper bound capacity constraint of dispatchable generator g.
$\underline{\phi}_g^k$ Lower bound capacity constraint of dispatchable generator g at scenario k.
$\overline{\chi}^k$ Balancing with excess of wind at scenario k.
$\underline{\chi}^k$ Balancing with shortage of wind at scenario k.
$\overline{\psi}_l$ Upper bound capacity constraint of wind turbine l.
$\underline{\psi}_l^k$ Lower bound capacity constraint of wind turbine l at scenario k.

I. Introduction

Renewable energy sources, in particular wind and solar, are seen as key technologies for mitigating climate change. Different economic instruments such as Green Targets and Certificates, Feed-in Tariffs or Feed-in Premiums have been introduced to support the development of these technologies. These instruments raise interesting economic questions [1]–[4], but can also sometimes lead to unintended consequences [5]–[7]. This paper assumes a feed-in premium incentive throughout; it very occasionally touches on an unintended effect, but otherwise does not discuss the relative merits of these economic instruments.

High rates of wind and solar generation in power systems also pose other, more micro economic and technical, challenges to market designers, system operators and generation firms [8]–[10]. Variability and unpredictability in case of high wind penetration may cause operational problems [11], [12] that need to be handled by operational flexibility in real time. We here refer to an European system with a balancing mechanism but no real time market, and refer to "flexibility reserve" as the reserve used in that balancing mechanism to manage unpredictability (we deal with variability in another paper [13]) in the sense of deviations with respect to wind forecast errors.

The determination of the optimal quantity and price of the flexibility reserve has been studied by several authors. In [14] a methodology that takes into account the uncertainty of the load and wind power forecasts and also the probability of generation failures is proposed to quantify the requirement for system reserve given a certain level of system reliability. This methodology is applied to the whole Irish power system as a case study. A common approach to deal with uncertainty is to rely on stochastic programming, as in [15] and [16]. The value of the operational flexibility is another important notion, which has been explored, for instance, in [17]. In [18] it is shown how to optimize the operation in a short-term forward electricity market using the concept of "stochastic security". Conditions for an optimal balancing market have been studied by several authors, who conclude that cost-reflectiveness with respect to the imbalance price is a key condition for a balancing market to be optimal [19], [20].

Generators act on both the day ahead and intraday markets as well as on balancing. Besides the literature on balancing, a number of strategies for optimal bidding in day ahead have been proposed in the academic literature. These strategies focus on uncertainty in the system and are also often based on stochastic programming. In [21], a model to derive the best offering strategy for a wind power producer is initially posed as a mixed-integer nonlinear program and finally reformulated and solved as an equivalent linear program. More microeconomic analysis of the behaviour of individual agents operating under uncertainty can be found in the literature such as in [22].

The use of stochastic programming for determining optimal offering strategies of wind power producers under uncertainty gives results more accurate than other methods that consider less information. This is shown in [23] and also in [24] where the model is posed as a two-stage stochastic program taking into account network constraints and a pool with a significant number of wind producers. In [25], optimal offers in quantity are calculated for wind power producers through a two-stage stochastic program in which the information on prices and available wind energy is incorporated as exogenous parameters using a scenario tree. The CVaR is used for representing risk averse behavior.

Other approaches include second-order cone programming as in [26], which is used to solve an optimal self-scheduling problem for a single firm based on a security constrained optimal power flow with risk aversion using a CVaR function.

The main contribution of this paper with respect to that literature can be summarized as follows. In contrast with most of the literature that poses the problem of wind accommodation through stochastic [15], [16] or robust [26] optimization, we state it in equilibrium terms with risk averse agents. We justify the departure from optimization on several grounds.

A first reason is that wind penetration is generally supported by instruments that distort the market away from perfect competition. This is the case of the feed-in premium which is the favorite EU proposal for reforming the support to renewable generation, as well as the feed-in tariff which has been the rule in many European countries. These distortions are incompatible with the perfect competition conditions implied by the pure optimization formulation of the short term power markets (e.g. two prices for the same product in a single market).

Equally relevant to justify the move to an equilibrium model, European systems contain idiosyncrasies that add distortions to the standard competition paradigms. One of them, treated here, implies non backward recursive payoffs in the sense that day ahead prices are also used in balancing, notwithstanding the state of the world in real time. This forward passing of price is the opposite of the backward mechanisms that state day ahead prices as expectations of real time prices in stochastic programming. This again departs from what can be done by optimization formulations.

Last, instruments such as feed-in premium or tariffs also generate risk exposures that can be quite different for conventional and wind generators. Barring a US type mechanism of the virtual bidding type that does not exist in Europe, these risks are not tradable. This requires a multi-agent setting where agents face different non tradable risks. This is again not amenable to a single agent representation (whether by stochastic, robust or risk function optimization) of the problem.

Summing up, we believe that the combination of balancing markets and wind policies idiosyncrasies introduces enough departures from the standard stochastic optimization paradigm to justify an extension to the more general equilibrium formulation at least when looking at the European market.

The balancing mechanism is thus central in this analysis. It can be modeled through econometric relations giving balancing prices as functions of energy or through fundamental representations that could include an unit commitment [27]. Because balancing is an evolving process subject to continuous discussion, we do not rely on an econometric representation but introduce a fundamental equilibrium model (without explicit unit commitment). This model can be adapted, sometimes at the cost of additional computational manipulation, to different market designs (e.g. different pricing schemes). This capability is also specific to equilibrium models and would not be possible in an optimization framework. This is discussed in more detail in Section II-D.

The content of the paper can be summarized as follows. We develop a short-term equilibrium-based model with high wind penetration where the wind producers benefit from a feed-in premium, and each generator solves a two-stage stochastic programming problem to optimize its profit. The Transmission System Operator (TSO) runs a day ahead market for reserve and a real time balancing mechanism. We use this set up to provide a first insight into the loss of competitiveness of conventional capacity that accompanied the penetration of wind in Europe. We examine the combined impact of feed-in premium and adequate pricing of the reserve on the revenue of conventional plants, and come up with contrasting results. The possibility to contract abundant reserve for flexibility close to real time confirms the loss of competitiveness of conventional plants. In contrast, flexibility reserves that need to be committed many hours in advance provide revenue that compensate the losses on the energy market.

The paper is organized as follows. In Section II the model attributes and main assumptions for the day ahead, reserve and balancing market are given. In Section III the mathematical formulation is described. An illustrative case study based on a stylized version of the Spanish power system is discussed in Sections IV and V. Conclusions close the paper.

II. Model Formulation

A. Assumptions

Electricity markets deal with wind integration through a wide range of organizations. The proposed model is inspired by the Spanish case but presents some general features that are common to all European markets and hence may also be of interest to other countries supporting renewable energy [28].

We consider a two settlements system with a day ahead energy market and a real time balancing mechanism. The Power eXchange (PX) clears the energy market in day-ahead under imperfect wind forecast. The TSO deals with deviations with respect to these forecasts in real time. It does so through reserves that it procures in a day ahead "market for reserve" and uses in real time[1] in "balancing". Flexibility reserves are of the upward and downward capacity type. Here we focus on the operation mechanism in an hour, and simplify the treatment by not explicitly considering any intra-day market.

Reserve and balancing deal with deviations with respect to scheduled quantities in day ahead. This can be represented by a standard two stage stochastic tree. We represent deviations with respect to forecast by scenarios of wind power output realization in one hour[2].The model is set up for a day ahead time horizon with an hourly time granularity.

Generators, consumers, and the TSO are the agents in the market. Generators operate dispatchable (conventional) generators with linear cost (without fixed operating cost) and wind turbines with zero generation cost. Deviations, up and down, are cleared by the TSO in real time within the reserve capacities procured in day ahead from generators. Consumers are represented by a linear inverse demand function in the day ahead energy market. They are charged a fraction of the socialized cost of the reserve capacity through an ex-post network charge that, for the sake of simplicity, but also not unrealistically, is not taken into account in the model (this assumes that consumers do not react to fixed connection charge in the short run). The government subsidizes scheduled wind generation through a premium that is financed by the general budget. The premium can be related to the spot price by a general function (the higher the spot price the lower the premium) but it is here taken as an exogenous parameter in

[1]In some electricity markets, like the Spanish market, there exists an intra-day market with intermediate auctions in the time between day ahead and real time; these auctions contribute to reduce the cost associated with the wind forecast error [21], [29].

[2]Deviations of demand and generation contingencies can be included in the scenario tree at the cost of additional technical developments. In order to simplify the presentation wind deviations constitute the only stochastic elements.

order to both simplify the presentation and allow for sensitivity analysis.

The model is posed as a two stage stochastic market equilibrium problem with risk averse generators. The first stage variables of the equilibrium model are the demand and the energy bids from conventional and wind generators in the energy market and the capacities in the flexibility reserve market. The second stage variables are those appearing in balancing and the auxiliary variables used to compute the risk function of the generators (a $CVaR$ involving the revenue from balancing). We make the blanket assumption that agents are price takers in the day ahead energy and reserve market as well as in the real time balancing market.

The energy market is typically modeled by a technology driven supply curve (marginal cost) and a linear demand function (marginal willingness to pay). The need for reserves is represented by a set of upper and lower bound constraints, that equate the supply and demand of that energy in balancing. These are discussed in detail in Sections II-B, II-C and II-D. Finally, network constraints are not considered in this model where the TSO is only involved in the organization of the reserve market and the balancing mechanism. Many of these features can be technically removed or modified to scale up the model or adapt it to other markets. The model is formulated as a complementarity problem.

B. Day ahead energy market and premium

The PX clears the energy market on an hourly basis in day ahead. Day ahead outcomes depend on an imperfect wind forecast but not on real time wind realization.

The energy market is modeled by a standard technology driven supply curve (marginal cost) and a linear demand function (marginal willingness to pay) represented by the inverse demand function

$$\rho = \rho_0 - \alpha_0 \cdot d_T \tag{1}$$

where ρ is the price for energy in day ahead and d_T is the total demand. The clearing of the day ahead energy market determines sales d_h, scheduled wind y_l and dispatchable generation x_g.

Scheduled wind generation y_l receives a feed-in premium ρ^+ (exogenous parameter), that adds to the equilibrium price in the generators' revenue. It is not directly paid by customers but charged to the general budget. Only the scheduled wind generation that is actually generated in real time receives the feed-in premium; the wind that was scheduled, but finally not generated, loses the feed-in premium in the balancing mechanism. No feed-in premium is paid to the wind energy that is generated in real time but was not scheduled.

In the Spanish market this premium was given until 12th July 2013, by a piecewise linear function with a floor and a cap, as described in [30]. We simplify the mechanism and assume a fixed premium ρ^+.

The contribution of the day ahead energy market to the revenue of firm f is

$$\sum_{l \in G_f} y_l \cdot (\rho^+ + \rho^*) + \sum_{g \in G_f} x_g \cdot (\rho^* - cg_g) \tag{2}$$

where ρ^* is the value of the price ρ, cleared by the energy market. $\sum_{l \in G_f} y_l \cdot (\rho^+ + \rho^*)$ is the revenue accruing to scheduled wind (feed-in premium and equilibrium price) and $\sum_{g \in G_f} x_g \cdot (\rho^* - cg_g)$ is the revenue collected by the scheduled dispatchable generation (equilibrium price minus generation cost).

C. Reserve Model

The TSO clears the market for committed flexible reserves on an hourly basis in day ahead. Different types of reserves are used in real power systems. It is convenient to classify them in two groups: i) those intervening for load-frequency control are usually very fast (seconds) and automatically operated; ii) those devoted to load-following operate on longer times (hours) and are part of the market [31]. This model concentrates on a load-following requirements due to wind that we call flexibility reserve; these imply ramping requirements and balancing for accommodating deviations from wind forecast. Flexibility reserve is procured by the TSO from dispatchable generators in day ahead and results in two products: capacities of committed upward (ru_g) and downward (rd_g) reserves in the day ahead.

In real power systems the requirement of flexibility reserves is usually based on deterministic and/or simple probabilistic approaches [32] with more than one criteria commonly used for sizing the load-following reserves. We here follow [33] and dynamically determine the required flexibility reserve on the basis of the generation scheduled from different units. We use the linear function $R_u = m_y \sum_{l \in G} y_l + m_x \sum_{g \in G} x_g$ of the scheduled generation to determine the upward reserve requirement R_u by the TSO. The coefficients m_y and m_x appearing in this relation, called balancing reserve factors, are exogenous and reflect the uncertain need for reserve associated with each technology. The whole set of constraints for reserve in the model is as follows (3)-(6):

$$\sum_{g \in G} ru_g \geq \underline{A} \cdot \left[m_x \cdot \sum_{g \in G} x_g + m_y \cdot \sum_{l \in G} y_l \right], \qquad (\overline{\kappa}), \tag{3}$$

$$\sum_{g \in G} rd_g \geq B \cdot \sum_{g \in G} ru_g, \qquad (\underline{\kappa}), \tag{4}$$

$$\sum_{g \in G} ru_g \leq \overline{A} \cdot \left[m_x \cdot \sum_{g \in G} x_g + m_y \cdot \sum_{l \in G} y_l \right], \qquad (\overline{\gamma}), \tag{5}$$

$$\sum_{g \in G} rd_g \leq \sum_{g \in G} ru_g, \qquad (\underline{\gamma}). \tag{6}$$

where (3) and (4) are the upper bound constraints for the upward and the downward reserve capacities respectively, and (5), (6) are the corresponding lower bound constraints. These constraints are inspired by the operation of real power systems, in particular the Spanish system.

The revenue accruing to firm f from committing flexibility reserve in the TSO day ahead reserve market is:

$$\sum_{g \in G_f} \left(ru_g \cdot (\overline{\kappa} + \overline{\gamma}) + rd_g \cdot (\underline{\kappa} + \underline{\gamma}) \right) \tag{7}$$

where $\overline{\kappa}+\overline{\gamma}$ (€/MW) is the shadow price of capacity for committed upward reserve, and $\underline{\kappa}+\underline{\gamma}$ (€/MW) is the shadow price of capacity for committed downward reserve.

D. Balancing

The balancing of real power systems must accommodate demand prediction errors, system contingencies, uncertainty of non dispatchable generation and forecast errors. We simplify the discussion by considering only wind power forecast errors, which are usually the largest demand for load following in the short term in system with high wind penetration [14]. The standard economic paradigm is that these services should be charged at marginal reserve cost to those that demand them, and paid at the same value to those that produce them. Other pricing mechanisms such as penalization factors on the market prices [9], [20] can be also applied.

Here, we consider a simplified and economically standard approach where the price and the amount of balancing energy from committed reserve result from a perfectly competitive market. The market is modeled as an optimization problem that minimizes the generation cost of balancing energy. This problem represents an auction organized by the TSO. From a modeling point of view, the result of this auction is integrated in the equilibrium through its KKT conditions. Let s_g^k and u_g^k be respectively the energy deployed from upward and downward flexible reserve ru_g and rd_g in real time. Let also z_l^k and v_l^k be the energy bought or sold by wind turbines in real time to compensate the discrepancy between what had been scheduled in day head and what could be delivered in real time. The auction is modeled, for each scenario k, as the following optimization problem:

$$\underset{s_g^k,u_g^k}{\text{minimize}} \sum_{g\in G}\left(s_g^k\cdot cg_g - u_g^k\cdot cg_g\right) \tag{8}$$

$$\text{subject to: } \sum_{g\in G} s_g^k = \sum_{l\in G} z_l^k, \qquad (\overline{\pi}^k) \tag{9}$$

$$\sum_{g\in G} u_g^k = \sum_{l\in G} v_l^k, \qquad (\underline{\pi}^k) \tag{10}$$

$$s_g^k \le ru_g, \qquad (\overline{\eta}_g^k) \tag{11}$$

$$u_g^k \le rd_g, \qquad (\underline{\eta}_g^k) \tag{12}$$

where the variables are s_g^k and u_g^k, and the fixed input data are the committed upward ru_g and downward rd_g reserve determined in real time, as well as the real time demand and supply of energy z_l^k and v_l^k by wind units. The constraints are the balancing between the energy used by wind turbines and energy provided by dispatchable generators for downward reserve (9), and upward reserve (10). Each unit can provide balancing energy only from the capacity that has been previously committed in the reserve market (11), (12). The objective function (8) minimizes the generation cost from reserves using the committed generators with the minimum generation cost for upward reserve and those with the maximum cost for the downward reserve to supply the energy from reserves.

The effect of balancing on revenue and cost differs depending on whether wind realization is short or in excess. This is taken into account by the balancing cost for firm f at scenario k given by the recourse function Q_f^k:

$$\begin{aligned} Q_f^k = &\sum_{g\in G_f}\left[s_g^k\cdot(cg_g-\overline{\pi}^k)+u_g^k\cdot(\rho^*-cg_g-\underline{\pi}^k)\right]+ \\ &+\sum_{l\in G_f}\left[z_l^k\cdot(\rho^+ +\overline{\pi}^k)+v_l^k\cdot(\underline{\pi}^k-\rho^*)\right] \end{aligned} \tag{13}$$

where ρ^* is the energy price on the day ahead market and $\overline{\pi}^k$ and $\underline{\pi}^k$ are the energy prices from committed upward and downward reserve respectively from (9) and (10).

In case of wind shortage, the cost incurred by wind turbine l that is short by z_l^k in scenario k is $z_l^k\cdot(\overline{\pi}^k+\rho^+)$: The plant loses the premium for the energy committed in the day ahead market and has to substitute it by conventional energy in balancing at the equilibrium price of committed upward reserve. On the other hand, the cost of a dispatchable generator g for providing the upward reserve energy s_g^k is $s_g^k\cdot(cg_g-\overline{\pi}^k)$, which is the unitary generation cost cg_g minus the price of the committed upward reserve $\overline{\pi}^k$.

In case of wind in excess, a wind turbine can sell an amount v_l^k in excess on its scheduled generation (ranging from zero to the available excess). Its revenue is $v_l^k\cdot(\rho^*-\underline{\pi}^k)$, which involves the unitary cost of downward reserve energy $\underline{\pi}^k$ and the revenue from selling the energy at the equilibrium price ρ^*. Alternatively, the cost incurred by the dispatchable generator g in scenario k to down u_g^k from its committed downward reserve to incorporate the additional wind is $u_g^k\cdot(\rho^*-cg_g-\underline{\pi}^k)$. The unitary cost is the equilibrium price ρ^*, which was already incorporated in the day ahead revenue, from which one subtracts the generation cost cg_g, minus the non generated energy and the price of downward balancing energy $\underline{\pi}^k$.

Summing up the balancing model uses a two-price settlement with one price for the upward energy and other price for the downward energy. The wind turbines do not lose the equilibrium price in balancing but they do not get the premium for quantities that they scheduled in day ahead and do not deliver in real time.

E. Scenario tree

The output of wind turbines is the only uncertainty considered in the model. It is represented by a scenario tree where each branch stands for a realization of the wind power output for one hour.

We use a Beta distribution to model the wind power forecast error [34], [35]. Let $q=\frac{\text{Power output}}{\text{Rated power}}\in[0,1]$ be the load factor for wind generation, [34] and [35] argue that this load factor fits a Beta distribution:

$$f(q)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}q^{\alpha-1}(1-q)^{\beta-1}, \quad q\in[0,1] \tag{14}$$

where $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$ is a scale factor such as $\int_0^1 f(x)dx=1$, and the parameters α and β are directly related to the mean (μ) and the standard deviation (σ) of the distribution:

$$\alpha=\mu^2\frac{1-\mu}{\sigma^2}-\mu, \qquad \beta=\alpha\left(\frac{1}{\mu}-1\right) \tag{15}$$

The analysis of empirical data shows that σ properly fits a linear function of μ, $\sigma = k_1 \cdot \mu + k_2$, [34], [35], [36] where the coefficients k_1 and k_2 mainly depend on the time horizon and the geographic dispersion of the wind turbines. Here we use the expression given in [36] for a time horizon of 24 h and large scale generation (normalized by the wind capacity installed):

$$\sigma = \frac{1}{5}\mu + \frac{1}{50} \quad \text{(in per unit)} \tag{16}$$

To build the scenarios, we divide the range $[0, 1]$ for the load factor into segments and associate each scenario with a segment. Let n be the number of scenarios, k the index for scenario and $z_k \in [0, 1]$, then the range $[0, 1]$ is discretized using $n + 1$ points, $0 = z_1 < z_2 < \ldots z_{n+1} = 1$. The value and the probability of scenario k are respectively:

a) $\mu(k) = \int_{z_k}^{z_{k+1}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha}(1-x)^{\beta-1} dx$, which is the expected value on the segment that defines the scenario.

b) $pr(k) = \int_{z_k}^{z_{k+1}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} dx$, which is the integral of the probability density function of the Beta distribution on the segment associated with the scenario.

The rule to select the points $0 = z_1 < z_2 < \ldots < z_{n+1} = 1$ is to get segments of equal probability: $\frac{1}{n} = \int_{z_k}^{z_{k+1}} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} dx$, $k = 1, 2, \ldots, n$.

III. Model equations

The model is cast as a complementarity problem, that consists of three groups of equations (the whole set of equations (30)-(48) is included in the appendix):

1) The global constraints (3)-(6) are the reserve constraints described in Section II-C. They represent the reserve capacity constraints of the market cleared by the TSO. Generators sell in this market at the prevailing clearing prices. These constraints are not included in the firms' KKT conditions but their dual variables appear in the objective functions of the firms.
2) The KKT conditions (34)-(39) represent the minimization of generation cost for balancing energy (8)-(12) as described in Section II-D.
3) The KKT conditions (40)-(48) for the problem of each firm, which is represented by (17)-(29) and described next. The optimization problem for each firm includes only the conditions for which the firm can make explicit decisions, taking market prices (energy, committed reserve capacity, ramping contracts and balancing energy) as given.

Each firm f solves a two stage stochastic program that represents the optimization of its operations between day ahead and balancing. The objective function is a risk adjusted cash flow $E-CVaR[Profit_f^k] = (1-\lambda_f) \cdot E[Profit_f^k] + \lambda_f \cdot CVaR[Profit_f^k]$ that the firm wants to maximize, where $E[\square] = \sum_k \text{Pr}^k \cdot \square$ is the expectation over the scenarios, $\lambda_f \in [0, 1]$ is the level of risk aversion ($\lambda_f = 0$ is risk neutral) and $Profit_f^k$ is the net profit for firm f at scenario k. $Profit_f^k = P_f - Q_f^k$ where P_f is the net profit of firm f in the first stage, that does not depend on the scenarios, and Q_f^k is the value of the recourse function (13).

Taking into account the previous definitions we can write the objective function (17) as $P_f - (1-\lambda_f) E[Q_f^k] - \lambda_f CVaR[Q_f^k]$. Where $CVaR[Q_f^k] = \zeta_f + \frac{1}{1-\theta_f} \sum_{k\in\Omega} \text{Pr}^k \cdot Q_f^{k,-}$ is the application to the recourse function of the standard definition of the CVaR given in [37].

The net profit in the first stage P_f is the sum of the income from energy sales $(\rho_0 - \alpha_0 \cdot \sum_{h\in F} d_h) \cdot d_f$, committed reserve $\sum_{g\in G_f} (ru_g \cdot (\overline{\kappa} + \overline{\gamma}) + rd_g \cdot (\underline{\kappa} + \underline{\gamma}))$, feed-in premium to wind generation $\sum_{l\in G_f} y_l \cdot \rho^+$, minus the cost of dispatchable generation $\sum_{g\in G_f} x_g \cdot cg_g$, all accruing in day-ahead. The optimization problem for each firm f is:

$$\begin{aligned}
\max_{\substack{d,x,y,z,rd,\\ ru,s,u,v,\zeta,Q^-}} \Bigg\{ & (\rho_0 - \alpha_0 \cdot \sum_{h\in F} d_h) \cdot d_f + \sum_{l\in G_f} y_l \cdot \rho^+ \\
& + \sum_{g\in G_f} (ru_g \cdot (\overline{\kappa} + \overline{\gamma}) + rd_g \cdot (\underline{\kappa} + \underline{\gamma})) \\
& - \sum_{g\in G_f} x_g \cdot cg_g - (1-\lambda_f) \cdot \sum_k \text{Pr}^k Q_f^k \\
& - \lambda_f \cdot \left(\zeta_f + \frac{1}{1-\theta_f} \sum_{k\in\Omega} \text{Pr}^k \cdot Q_f^{k,-} \right) \Bigg\}
\end{aligned} \tag{17}$$

subject to:

$$x_g + ru_g \le \overline{X}_g, \qquad (\overline{\phi}_g) \tag{18}$$
$$x_g + s_g^k - rd_g \ge \underline{X}_g, \qquad (\underline{\phi}_g^k) \tag{19}$$
$$y_l \le \overline{Y}_l^{\max}, \qquad (\overline{\psi}_l) \tag{20}$$
$$y_l - z_l^k \ge \underline{Y}_l^k, \qquad (\underline{\psi}_l^k) \tag{21}$$
$$\sum_{g\in G_f} x_g + \sum_{l\in G_f} y_l = d_f, \qquad (\nu_f) \tag{22}$$
$$z_l^k \le \overline{Y}_l^{max} - \overline{Y}_l^k, \qquad (\overline{\iota}_l^k) \tag{23}$$
$$y_l - z_l^k \le \overline{Y}_l^k, \qquad (\underline{\iota}_l^k) \tag{24}$$
$$v_l^k \le \overline{Y}_l^k, \qquad (\xi_l^k) \tag{25}$$
$$v_l^k + y_l - z_l^k \le \overline{Y}_l^k, \qquad (\tau_l^k) \tag{26}$$
$$ru_g \le \overline{R} \cdot (\overline{X}_g - x_g), \qquad (\overline{\delta}_g) \tag{27}$$
$$rd_g \le \underline{R} \cdot (\overline{X}_g - x_g), \qquad (\underline{\delta}_g) \tag{28}$$
$$\zeta_f + Q_f^{k,-} \ge Q_f^k \qquad (\sigma_f^k) \tag{29}$$

where $d, x, y, z, rd, ru, s, u, v, Q_f^{k,-}$ are positive variables.

Equations (18)-(29) comprise upper and lower bounds for dispatchable plants (18), (19), wind turbines (20), (21), energy balancing in day ahead (22), upper and lower bounds for the purchases of wind turbines due to shortage of wind (23), (24), upper bound for wind generation in excess (25), energy balance for wind turbines (26), ramping constraints for the upward (27) and the downward (28) reserve, and finally the auxiliary constraint for the CVaR calculation (29). The model is implemented and solved using the software PATH in GAMS [38].

IV. Case Study and Discussion

The model is applied to a stylized version of the Spanish electricity system. It can, in principle, be used to study the impact of a number of factors present in real systems such as:

1) Premium to wind generation (policy).
2) Expected wind forecast (physical characteristic of the system).
3) Risk aversion of the firms (market characteristic).
4) Wind power forecast error distribution (physical characteristic of the system).
5) Reserve requirement set by the TSO.
6) Level of energy demand compared to the installed power capacity (market characteristic).
7) Mothballing of existing power plants (company decision subject to regulatory approval).
8) Pricing scheme for capacity of balancing reserve (market design).
9) Pricing scheme for energy from balancing reserve (a market design).
10) Number of firms (market structure).
11) Technical capacity for ramping (technology characteristic).
12) Scheme of the balancing mechanism (market design).
13) Arbitrage between day ahead and balancing (market characteristic).

We illustrate this potential by an analysis of three questions leading to a more in depth investigation of the demand for flexibility reserve. Even though the model is designed for several firms operating in the market, it is only used here with a single company to simplify the discussion.

A. Case Data

The case study only considers wind and three conventional non hydro technologies: Combined Cycle Gas Turbine (CCGT), Coal and Nuclear. The conventional technologies are the only ones assumed dispatchable. Generators' data are listed in Table I; capacities, number of units and year of construction are taken from [39], generation cost and ramping capabilities come from [40]. The system has a total installed capacity of 66105.49 MW of which 22573.00 MW (34.15%) corresponds to wind and 43532.49 MW to dispatchable generation. We assume that the CCGT plants with the highest cost are mothballed and that 50% of the CCGT capacity with the lower cost is also mothballed. The resulting system still have a CCGT capacity of 10632.7 MW with a generation cost of 43.45 €/MWh. It must be noticed that CCGT are often the marginal plants in the following case studies. They then set the day ahead price at their marginal cost while making a zero margin.

Bounds and weights take the following values: $\underline{A} = 0.9$, $\overline{A} = 1.1$ (committed upward reserve in between $90\% - 110\%$ of the TSO requirement), $B = 0.4$ (committed downward reserve at least 40% of committed upward reserve), $m_y = 0.15$ (15 % balancing reserve factor for wind), $m_x = 0.02$ (2% balancing reserve factor for dispatchable generation), and confidence level $\theta_f = 0.95$ for $CVaR_\theta$.

The parameters ρ_0 and α_0 of the inverse demand function have been calculated by stating a price elasticity of -0.3 (data from [41]) at a reference point $(\overline{\rho}, \overline{d})$ where $\overline{\rho} = \rho_0 - \alpha_0 \cdot \overline{d}$. Taking $\overline{\rho} = 48.42$ €/MWh and $\overline{d} = 28753.32$ MWh that correspond to hourly average for the Spanish system in 2012, one obtains $\rho_0 = 209.82$ €/MWh and $\alpha_0 = 0.0056133$ €/(MWh)^2.

TABLE I: Summary of generator data

Technology	Net Capa. $\overline{X}_g$ (MW)	N. of units	Aver. age (years)	cg_g (€/MWh)	$\overline{R}_g = \underline{R}_g$ % of $\overline{X}_g$
CCGT	4395.86	10	10.7	45.82	53.33
CCGT	21265.40	43	6.2	43.45	53.33
Nuclear	1519.23	2	32.2	10.91	2.08
Nuclear	6053.35	6	28.0	10.29	2.08
Coal	2035.89	4	25.1	37.50	20.00
Coal	5119.13	18	34.8	38.44	25.00
Coal	1198.12	4	34.8	19.77	25.00
Coal	1945.51	5	34.8	20.24	25.00
Wind	22573.00	-	-	-	-
Total	**66105.49**				**16315.53**

Uncertainty is modeled through a scenario tree with 12 branches that represent the power output of wind turbines. Scenarios are constructed for a given value of the expected wind μ, according to the methodology and assumptions described in Section II-E. Four examples of scenario tree for different wind forecasts μ are listed in Table II. All these scenarios have the same probability ($\frac{1}{12} \approx 8.33\%$); this results from the particular discretization of the original Beta distribution as discussed in II-E. Notice that this does not assume an uniform error distribution. One shall note that the variability of wind generation is usually in the range $[0.4\mu, 1.6\mu]$ except for very low values of expected wind (see Table II).

TABLE II: Data of the scenario tree for wind

Scenario	Normalized power output (%)[1]			
1	0.59	12.64	26.42	35.55
2	1.20	16.13	33.34	46.56
3	1.69	18.12	37.13	52.44
4	2.15	19.76	40.16	56.99
5	2.62	21.24	42.83	60.89
6	3.12	22.66	45.34	64.44
7	3.66	24.09	47.82	67.81
8	4.27	25.60	50.35	71.13
9	5.00	27.27	53.08	74.52
10	5.93	29.24	56.19	78.18
11	7.27	31.86	60.16	82.46
12	10.57	37.34	67.60	89.05
μ (%)[1]	4.01	23.83	46.70	65.00
σ (%)[1]	2.73	6.66	11.20	14.83
max. (base μ)	2.64	1.57	1.45	1.37
min. (base μ)	0.15	0.53	0.57	0.55

Given this input data, the resulting equilibrium problem has a size of 645 variables, and takes an average time of 1.499 seconds to be solved with GAMS/PATH running on a laptop with an Intel CORE i7 processor and 6 GB RAM. The average computation time per case has been calculated on a sequence of 1681 runnings with hot start. Without hot start the solving time is ~ 4 seconds for each problem.

B. Problem statement

The price of energy in day ahead is a particularly relevant output of markets with high wind penetration. Similarly, the expected and risk adjusted gross margins made by the conventional and wind technologies are crucial parameters in the decision to invest today in the European market. We illustrate the use of the model by concentrating on the impact of five main factors on these elements:

a) The forecast wind level: High wind power tends to

[1]Calculated over the wind capacity installed.

reduce the equilibrium price. This is a direct consequence of wind replacing the most expensive dispatchable generators. It directly affects the margins made by conventional and wind plants. Table III (columns 2 and 3) reports results for average feed-in premium to wind generation and risk aversion but with different wind forecast μ. As expected, higher wind forecast results in higher scheduled wind and lower scheduled conventional generation. The global effect is a significant decrease of the equilibrium price of 17% that combines to the lower scheduled conventional generation in day ahead. At the same time, the higher scheduled wind generation also increases the capacity available for flexibility reserve, which reduces the revenue from reserve in day ahead. The end result is a drastic loss of profits of dispatchable generators (see row "1st stage dispatchable generators" in Table III) in the day ahead market. Revenue from balancing only very partially compensate for that loss (row "2nd stage dispatchable generators" in Table III). Overall, this drastic drop of revenue decreases the incentive to invest in conventional plants and increases the incentive to mothball or even dismantle existing capacities.

b) The feed-in premium is analyzed subject to average wind condition and risk aversion. Columns 4 and 5 of Table III reveal that the premium does not significantly affect the day ahead equilibrium even if it induces a small increase of scheduled wind. The high premium tends to enhance wind scheduling in day ahead but the loss of this premium in balancing, when wind generators are short, makes the payoff more risky, which has an impact on the risk adjusted profit at the second stage. Increasing the feed-in premium to wind generation obviously increases wind revenue, but this seems to take place without distorting the day ahead market. It is well known however that this efficiency result only holds where wind generation remains sufficiently low compared to demand, so that the electricity price remains positive (see [7] for a study of the distortion in the short run of a market with premium and high wind penetration).

c) The increased risk aversion is examined with average wind condition and feed-in premium to wind generation. One finds from columns 6 and 7 in Table III that an increase of the level of risk aversion λ of the firms only has a low impact on the short run market. The price in the day ahead market remains at 43.45 €/MWh that corresponds to the high capacity of the CCGT units with this operating cost. In contrast with the effect of a higher premium, a higher risk aversion decreases scheduled wind, albeit not by a large amount. The bad news is the decreasing margins of both types of plants. Conventional plants produce more than in the case of high wind forecast, but make the same margin due to the fact that this additional generation comes from the same CCGTs that make zero margin when they are marginal. Because of the reduction of activity in balancing resulting from the risk aversion, these plants also see their profits reduced on that market. The conclusion of these results is that risk aversion further reduces the incentive to invest.

d) Wind induced demand for flexibility reserve. Except for column 2 with low wind and hence most fossil capacity committed for generation, none of the above cases shows any constraint on flexibility reserve in day ahead; the consequence is the lack of revenue from the day ahead reserve market. Wind increases the demand for reserve but at the same time makes existing capacity available for that reserve. This lack of revenue from flexibility reserve, combined with the lower price in the merit order due to wind generation, is the mechanism that eventually leads to mothballing and possibly dismantling dispatchable capacities. The following analyzes this mechanism in more detail by considering two situations that differ by the demand for reserve and its impact on energy prices. We motivate the analysis by sourcing it in the error on wind forecast over different horizons. An alternative motivation, not discussed here, can be made by invoking ramping requirement in more detail than our equations (27) and (28).

i) Suppose first that a forecasting period of 6 hours or less is sufficient for committing flexibility reserve, and assume that we can move the gate closure of the energy and reserve markets to that horizon (through some intraday market that we do not characterize otherwise). Assume that wind forecast is relatively accurate over that horizon. The residual error between the forecast and the realization can be characterized by a standard deviation σ. This defines a range for the possible wind power output values that is relatively narrow, $[\mu - \sigma, \mu + \sigma]$, where μ is the expected value for wind and usually $\sigma < \frac{1}{5}\mu$.
ii) Alternatively consider a forecasting period of 24 hours, typical of an unit commitment problem, as necessary for committing flexibility reserve. We are then in our generic day ahead/real time context and now need to work with the Beta distribution discussed in section II-E. In case of a forecast error with mean μ and standard deviation $\sigma = \frac{1}{5}\mu + \frac{1}{50}$, (normalized values with respect to the installed wind capacity), the range of uncertain values is typically $[0.4\mu, 1.6\mu]$. This implies taking a domain of approximately $\pm 60\%$ over the expected value μ to cover most of the uncertainty.

Depending on the amount of available reserve, the committed reserve, the expected wind and the variability of wind, a number of different behaviors have been observed in the model. We discuss two configurations in this section and list other situations in the following:

1) Consider first the case where demand for flexibility reserve is sufficiently small that it never constraints scheduled wind: (scheduled wind) $-$ (lowest wind scenario) $\leq$ (committed upward reserve). This situation occurs when the uncertainty on wind generation is small, something that occurs when one can move the gate closure close to real time. Wind forecast is then good and balancing can take advantage of a slack reserve market. The observation shows that this is the situation where the premium ρ^+ has little effect on the amount of scheduled wind. This is what happens in the case depicted in columns 4 and 5 of Table III.
2) Consider now the situation where the demand for flexibility reserve can constrain scheduled wind. This occurs in situations where a given value of the premium (that could be zero) leads to an amount of the scheduled wind that hits, at least the constraint (scheduled wind) $-$

TABLE III: Simulation results for different configurations

	$m_y = 0.15$ (wind), $m_x = 0.02$ (dispatch. gen.)						$m_y = 0.60$, $m_x = 0.02$	
	Case 1		Case 2		Case 3		Case 4	
μ (%) expected wind	4.01	65.00	23.83	23.83	23.83	23.83	23.83	23.83
ρ^+ (€/MWh) premium	30.00	30.00	0.00	80.00	30.00	30.00	0.00	80.00
λ risk aversion (0 is risk neutral)	0.40	0.40	0.40	0.40	0.00	1.00	0.40	0.40
Energy demand day ahead (MWh)	27988.89	29639.08	29639.08	29639.08	29639.08	29639.08	27667.11	27717.02
Scheduled wind generation (MWh)	873.91	9696.58	3839.26	3850.16	4088.30	3548.96	4578.05	7012.83
Scheduled dispatch. gen. (MWh)	27114.99	19942.50	25799.82	25788.92	25550.78	26090.12	23089.06	20704.18
Equilibrium price (€/MWh)	52.71	43.45	43.45	43.45	43.45	43.45	54.52	54.24
Reserve requirement (MW)	673.39	1853.34	1091.89	1093.30	1124.26	1054.15	3208.61	4621.78
Upward reserve committed (MW)	740.72	1671.87	986.04	996.94	1235.07	1159.56	2887.75	4159.60
Downward reserve commit. (MW)	590.69	668.76	395.94	398.79	1235.07	463.83	1162.93	1673.49
Max. available reserve (MW)	740.72	4565.81	1442.10	1447.92	1574.92	1287.29	2887.75	4159.60
INCOMES (€):								
Energy sales in day ahead	1475320.88	1287758.71	1287758.70	1287758.71	1287758.71	1287758.71	1508332.91	1503289.16
Premium to scheduled wind	26217.14	290897.26	0.00	308012.95	122648.97	106468.93	0.00	561026.53
Supply of upward reserve capacity	12865.72	0.00	0.00	0.00	0.00	0.00	59938.22	84151.84
Supply of down. reserve capacity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
COSTS (€):								
Cost of dispatchable generation	816662.92	505032.92	759521.40	759047.89	748701.31	772134.33	641744.35	538126.42
Contribution of the $\beta_f \cdot CVaR[Q]$	21758.83	49106.75	17132.17	49217.49	0.00	64999.49	29968.62	205372.92
Contribution of the $(1-\beta_f) \cdot E[Q]$	10662.40	-9841.11	-5713.64	-874.05	-15109.68	0.00	-9038.36	127398.48
PROFIT (€)								
Total (wind + dispatch.)	679970.16	1090024.90	537760.04	838180.53	676816.04	632872.83	941590.72	1398010.31
1st stage wind turbines	72281.48	712194.06	166808.32	475294.79	300277.37	260664.32	249582.58	941381.82
2nd stage wind turbines	-16886.44	-22744.77	-6950.69	-21266.15	-41874.72	-3695.82	-12996.07	-215420.27
1st stage dispatchable generators	625459.34	361428.99	361428.98	361428.99	361428.99	361428.99	676944.20	668959.29
2nd stage dispatchable generators	-884.22	39146.63	16473.43	22722.90	56984.40	14475.34	28060.00	3089.47

(lowest wind scenario) $\leq$ (committed upward reserve). This occurs when the uncertainty on wind generation is high, for instance because of the need to commit the reserve well in advance. This can be due to a lack of intraday market or to the impossibility to trade capacity in intraday (e.g. in a continuous intraday energy market without intermediate auction). In this case the premium ρ^+ to wind generation may have a strong impact on the amount of scheduled wind.

We analyze this situation in columns 8 and 9 of Table III. The two cases refer to the average wind forecast ($\mu = 23.83\%$) and the average risk aversion ($\lambda = 0.4$); its characteristic is to implicitly assume a longer forecast horizon for wind implying $m_y = 0.6$ (in contrast with $m_y = 0.15$ otherwise), while m_x remains at the value 0.02 used in cases 1 to 3. In short the demand for reserve for load following due to wind is higher. We focus on the impact of changing the feed-in premium to wind generation.

In contrast with case 2, the increase of the feed-in premium to wind generation now implies a significant increase of the scheduled wind generation. But also in contrast with case 2, the two feed-in premium to wind generation cases lead to an increase in the price of electricity compared to those found in all other cases (from 1 to 3). This does not comply with the common wisdom that explicitly sees wind increase as implying a decrease of the electricity price. The justification is to be found in the higher demand for flexibility reserve implied by the longer horizon necessary for forecasting wind production and committing reserve (moving m_y from 0.15 to 0.6). One indeed observes higher upward and downward committed reserves in case 4 compared to all other cases, and an increase of these reserve inside case 4 when the feed-in premium to wind generation increases. The end result is a revenue from committed reserve in the day ahead market that restores the profit accruing to dispatchable generators in day ahead to a value obtained in the case of low wind as seen by comparing the rows "1st stage dispatchable generators" in columns 8 and 9 to the same information in column 2 (Table III).

e) Other cases of high demand for reserves: The following circumstances, not further discussed here, can also lead to tight constraints on reserve that, if properly priced, could restore the profits of conventional generators and hence stop the incentive to mothball of dismantle.

i) Dispatchable generators may not have enough capacity to satisfy simultaneously the requirement for energy demand and reserve commitment. Firms arbitrage between using the generation capacity of dispatchable generators for reserve or for generation of energy in day ahead. This will happen at the end of a mothballing, dismantling or insufficient investment process. It will induce capacity cycles.
ii) Committed reserve hits technical limits. This depends on the overall flexibility of the remaining dispatchable plants.
iii) Wind incentive can have unintended effects. Consider a situation where the requirement for flexibility reserve in day ahead by the TSO is sufficiently high that it does not constraint wind generation over its whole range of uncertainty (e.g. $m_y = m_x = 0.3$). Suppose also that generation capacity is tight with respect to demand (for instance as a result of mothballing). A high enough feed-in premium (for instance $\rho^+ = 80$ €/MWh) can make it profitable to supply more demand by scheduling wind even in excess of the highest scenario, at the same time as moving dispatchable generation into flexibility reserve. The incentive for this strategy has two origins.

It first results from the premium accruing from total wind generation in day ahead compensating the cost of energy purchased on balancing in case of wind shortage. The second incentive results from the higher demand for reserve flexibility by the TSO that also increases the price of energy. The strategy is only a consequence of the market design (in this case the high feed-in premium) and does not involve any exercise of market power.

V. Final remarks

It maybe useful to note at this stage that the above model is designed to accommodate several firms and hence cannot in principle be restated as an optimization problem. One can further remark that the formulation of the profit of the firm with the equilibrium prices intervening both in day ahead and balancing also differs from a (stochastic) optimization problem even in the single firm case. Invoking a day ahead energy price in revenues occurring in balancing, (which seems common in Europe), is indeed incompatible with the backward recursive structure of stochastic optimization and requires an equilibrium formulation. It is thus conjectured that this more general formulation will allow for the study of various imperfections of the market designs in the implementation of renewable policies that are out of reach of optimization models.

The different case studies illustrate the common idea that the "devil is in the detail". The first case study confirms, the now well admitted idea, that wind penetration reduces the revenue of conventional plants to the point that investment is stopped and plants necessary for the system may be withdrawn from operations because of insufficient revenue. The second and third cases suggest that this intuitive result is robust as one essentially observes the same losses of revenue for quite different structuring parameters (feed-in premium and risk aversion). The fourth case study suggests that the situation might be more complex: The change of a single apparently innocuous coefficient (increasing m_y from 0.15 to 0.6) can completely modify important policy conclusions. The analysis indeed shows that conventional plants that are necessary for the operations of the system, in this case to provide flexibility reserve, will remain in the system even in case of high wind penetration if their services are economically (at marginal cost) priced. This requires that one is willing to acknowledge the demand for these services and the need to remunerate those that provide them. Referring to current European discussions on adequacy, it is also useful to note that this pricing scheme can be embedded in energy only markets (hence without requiring a capacity market) and will not keep redundant conventional plants on line (it does not contain any element of "State Aid").

From a more technical point of view, insights on how different factors affect the day ahead energy price can be obtained by considering the KKT conditions of the market model. Specifically one can observe that the energy price is:

a) Reduced by the feed-in premium, the availability of downward reserve (the more the committed downward reserve the better), and the must-run capacity of wind turbines. This latter parameter is fixed to zero in this paper, but it could be made greater than zero for technical reasons.
b) Increased by poor wind conditions with respect to the system capacity, a lack of committed downward and upward reserve, shortage of wind in real time compared to the scheduled wind generation, dispatchable generation cost, and insufficient dispatchable generation capacity.

An analysis of the impact of different factors on plant revenue requires some other development that will be taken up in a future paper.

VI. Conclusion

We propose a stochastic equilibrium model to investigate the impact of market design on the remuneration of conventional and wind generation plants in a market with potentially high penetration of renewable. The justification for resorting to an equilibrium instead of an optimization approach even without market power is twofold. A first reason is that the market is now composed of different firms that trade energy and services; the equilibrium formulation is most natural for this situation. The second, possibly more compelling reason is that renewable policies are implemented through different market instruments that cannot necessarily be cast in an optimization form. The proposed model is inspired by the Spanish situation but the approach is general. We try to show that the approach can embed general features of market design (a two settlement system), market idiosyncrasies (a somewhat detailed representation of the balancing market) and general economic characteristics of agents like risk aversion. Like stochastic programming, equilibrium models are amenable to a treatment of uncertainty.

We illustrate the use of the model by focusing on a timely European question, namely the revenue accruing to conventional plants in market with high renewable penetration. The common wisdom (and the observation of the market) is that renewable induce a decrease of energy prices together with a reduction of the activity of the conventional units, and hence an overall decrease of their revenue that put their sustainability in the market in question. We use the model to illustrate that this phenomenon indeed seems rather stable under different structural assumptions (same orders of magnitude in the loss of revenue for quite different assumptions of wind premium and risk aversion). But we also show that the phenomenon may also crucially depend on the demand for ancillary services (here frequency control) induced by renewable and on their pricing by the market design. Both subjects are contentious: The incremental demand of reserve due to renewable is not well understood [14], [16], [27], [36], [42]–[45] and the pricing of ancillary services is as diverse as the number of power systems in Europe. We model the demand for reserve through a coefficient and adopt an elementary, but sound, economic principle for pricing both reserve capacity and energy.

The question of how the capacity payments, "missing money", is determined in systems with high wind penetration (comparing regulation and deregulation) is also studied in [6], and they get similar conclusions to those summarized here. We find that a higher demand for load following reserve and an economically sound pricing (marginal cost pricing) restore the revenue of the conventional plants. The question of the sustainability of conventional plants then boils down to the proper identification of the demand for services (taking into account their provision by wind units themselves) and the acceptance that they be properly remunerated.

Using the framework described in this paper we plan as future works to consider situations with several firms with a separation of wind turbines and dispatchable generators, a complete optimization problem for the TSO (not only the minimization of generation cost from reserves), a comparison of different schemes for pricing and the balancing mechanism, and firms with different competitive behaviour (price makers, price takers). In our model the agents are price takers in both, day ahead and balancing market, a model with some points in common has been proposed in [46] with the wind turbines as price makers in balancing.

APPENDIX

Summary of the whole set of equations for the equilibrium problem. Let be $\mathcal{L}_f$ the Lagrangian function for firm f, $\mathcal{L}_B$ the Lagrangian function for the cost minimization problem for generation from reserve, and $\Lambda_f^k = (1-\lambda_f)\cdot \mathrm{Pr}^k + \sigma_f^k$ (to reduce the length of the expressions). The whole equilibrium problem consists of three groups of equations:

1) The global constraints:

$$\underline{A}\cdot\left[m_x\cdot\sum_{g\in G} x_g + m_y\cdot\sum_{l\in G} y_l\right] \le \sum_{g\in G} ru_g, \qquad (\overline{\kappa}), \tag{30}$$

$$B\cdot\sum_{g\in G} ru_g - \sum_{g\in G} rd_g \le 0, \qquad (\underline{\kappa}), \tag{31}$$

$$\overline{A}\cdot\left[m_x\cdot\sum_{g\in G} x_g + m_y\cdot\sum_{l\in G} y_l\right] \ge \sum_{g\in G} ru_g, \qquad (\overline{\gamma}), \tag{32}$$

$$\sum_{g\in G} rd_g - \sum_{g\in G} ru_g \le 0, \qquad (\underline{\gamma}) \tag{33}$$

2) The KKT conditions for the minimization of generation cost from reserves:

$$\frac{\partial\mathcal{L}_B}{\partial s_g^k} = cg_g - \overline{\pi}^k + \overline{\eta}_g^k \ge 0, \qquad \perp\ s_g^k \ge 0 \tag{34}$$

$$\frac{\partial\mathcal{L}_B}{\partial u_g^k} = -cg_g - \underline{\pi}^k + \underline{\eta}_g^k \ge 0, \qquad \perp\ u_g^k \ge 0 \tag{35}$$

$$\frac{\partial\mathcal{L}_B}{\partial\overline{\pi}^k} = \sum_{l\in G} z_l^k - \sum_{g\in G} s_g^k = 0, \qquad \overline{\pi}^k\ \text{free} \tag{36}$$

$$\frac{\partial\mathcal{L}_B}{\partial\underline{\pi}^k} = \sum_{l\in G} v_l^k - \sum_{g\in G} u_g^k = 0, \qquad \underline{\pi}^k\ \text{free} \tag{37}$$

$$\frac{\partial\mathcal{L}_B}{\partial\overline{\eta}_g^k} = s_g^k - ru_g \le 0, \qquad \perp\ \overline{\eta}_g^k \ge 0 \tag{38}$$

$$\frac{\partial\mathcal{L}_B}{\partial\underline{\eta}_g^k} = u_g^k - rd_g \le 0, \qquad \perp\ \underline{\eta}_g^k \ge 0 \tag{39}$$

3) The KKT conditions for each firm f (here only for primal variables):

$$\frac{\partial\mathcal{L}_f}{\partial d_f} = -(\rho_0 - \alpha_0\sum_{h\in F} d_h) - \nu_f \ge 0 \perp\ d_f \ge 0 \tag{40}$$

$$\frac{\partial\mathcal{L}_f}{\partial rd_g} = -\underline{\kappa} - \underline{\gamma} + \sum_{k\in\Omega}\underline{\phi}_g^k + \underline{\delta}_g \ge 0 \perp\ rd_g \ge 0 \tag{41}$$

$$\frac{\partial\mathcal{L}_f}{\partial ru_g^k} = -(\overline{\kappa}+\overline{\gamma}) + \overline{\phi}_g + \overline{\delta}_g \ge 0 \perp\ ru_g \ge 0 \tag{42}$$

$$\frac{\partial\mathcal{L}_f}{\partial v_l^k} = \Lambda_f^k\cdot(\underline{\pi}^k - \rho^*) + \xi_l^k + \tau_l^k \ge 0 \ \perp\ v_l^k \ge 0 \tag{43}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_f}{\partial x_g} &= cg_g + \overline{\phi}_g - \sum_{k\in\Omega}\underline{\phi}_g^k + \nu_f \\ &\quad + \underline{R}\cdot\underline{\delta}_g + \overline{R}\cdot\overline{\delta}_g \ge 0 \ \perp\ x_g \ge 0\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_f}{\partial y_l} &= -\rho^+ + \overline{\psi}_l + \sum_{k\in\Omega}(\tau_l^k + \underline{\iota}_l^k - \underline{\psi}_l^k) \\ &\quad + \nu_f \ge 0 \ \perp\ y_l \ge 0\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{\partial\mathcal{L}_f}{\partial z_l^k} &= \Lambda_f^k\cdot(\overline{\pi}^k + \rho^+) + \underline{\psi}_l^k + \\ &\quad + \overline{\iota}_l^k - \underline{\iota}_l^k - \tau_l^k \ge 0 \ \perp\ z_l^k \ge 0\end{aligned} \tag{46}$$

$$\frac{\partial\mathcal{L}_f}{\partial Q_f^{k,-}} = \frac{\lambda_f}{1-\theta_f}\mathrm{Pr}^k - \sigma_f^k \ge 0 \ \perp\ Q_f^{k,-} \ge 0 \tag{47}$$

$$\frac{\partial\mathcal{L}_f}{\partial\zeta_f} = \lambda_f - \sum_{k\in\Omega}\sigma_f^k = 0; \quad \zeta_f \quad \text{free} \tag{48}$$

REFERENCES

[1] National Round Table on the Environment and the Economy (Canada). *Economic Instruments for Long-term Reductions in Energy-based Carbon Emissions*. 2005.

[2] G. Cornelis van Kooten and G. R. Timilsina. Wind Power Development. Economics and Policies. Policy Research Working Paper WPS4686, The World Bank Development Research Group Environment and Energy Team, Washington, DC, March 2009.

[3] F. Borggrefe and K. Neuhoff. Balancing and Intraday Market Design: Options for Wind Integration. Discussion paper 1162, DIW Berlin, German Institute for Economic Research, 2011.

[4] International Renewable Energy Agency (IRENA). 30 Years of Policies for Wind Energy. Lessons from 12 Wind Energy Markets. Tech report, IRENA and Global Wind Energy Council (GWEC), C67 Office Building, Khalidiyah Street, P. O. Box 236, Abu Dhabi, United Arab Emirates, 2012.

[5] B.F. Hobbs. -\$300 Bid Floor Due to Renewable Subsidies?, Fix the Real Problem: Production-Based Incentives. *CAISO Market Surveillance Committee Meeting*, 18 March 2011.

[6] T. D. Mount, S. Maneevitjit, A. J. Lamadrid, R. D. Zimmerman, and R. J. Thomas. The Hidden System Costs of Wind Generation in a Deregulated Electricity Market. *The Energy Journal*, 33(1):161–186, January 2012.

[7] R. Baldick. Wind and Energy Markets: A Case Study of Texas. *Systems Journal, IEEE*, 6(1):27–34, 2012.

[8] B.F. Hobbs. Transmission planning and pricing for renewables: Lessons from elsewhere. In *Power and Energy Society General Meeting, 2012 IEEE*, pages 1–5, 2012.

[9] D. Newbery. Contracting for wind generation. Technical Report EPRG Working Paper 1120, Electricity Policy Research Group, Unversity of Cambridge, Austin Robinson Building, Sidgwick Avenue, Cambridge CB3 9DE, UK, July 2011.

[10] A.J. Conejo, J.M. Morales, and J.A. Martinez. Tools for the Analysis and Design of Distributed Resources - Part III: Market Studies. *Power Delivery, IEEE Transactions on*, 26(3):1663–1670, july 2011.

[11] J. Cox. Impact of Intermittency: How Wind Variability Could Change the Shape of the British and Irish Electricity Markets. Technical report, Pöyry Energy Consulting, King Charles House, Park End Street, Oxford OX1 1JD, UK, July 2009.

[12] G. Giebel, R. Brownsword, G. Kariniotakis, M. Denhard, and C. Draxl. The State of the Art in Short-Term Prediction of Wind Power. Technical Report aplus deliverable_D1.2_STP-SOTA_v1.0.docx, ANEMOS.plus and SafeWind projects, Frederiksborgvej 399, 4000 Roskilde, Denmark, 28 January 2011.

[13] S. Martin. *Stochastic-Based Models for Electricity Market Analysis with High Wind Energy Penetration*. Phd dissertation, Universidad de Málaga, 2014.

[14] R. Doherty and M. O'Malley. A new approach to quantify reserve demand in systems with significant installed wind capacity. *Power Systems, IEEE Transactions on*, 20(2):587–595, 2005.

[15] J.M. Morales, A.J. Conejo, and J. Perez-Ruiz. Economic Valuation of Reserves in Power Systems With High Penetration of Wind Power. *Power Systems, IEEE Transactions on*, 24(2):900–910, 2009.

[16] A. Papavasiliou, S.S. Oren, and R.P. O'Neill. Reserve Requirements for Wind Power Integration: A Scenario-Based Stochastic Programming Framework. *Power Systems, IEEE Transactions on*, 26(4):2197–2206, 2011.

[17] F. Bouffard and M. Ortega-Vazquez. The value of operational flexibility in power systems with significant wind power generation. In *Power and Energy Society General Meeting, 2011 IEEE*, pages 1–5, july 2011.

[18] F. Bouffard and F.D. Galiana. Stochastic Security for Operations Planning With Significant Wind Power Generation. *Power Systems, IEEE Transactions on*, 23(2):306–316, may 2008.

[19] C. Hiroux and M. Saguan. Large-scale wind power in European electricity markets: Time for revisiting support schemes and market designs? *Energy Policy*, 38(7):3135–3145, 2010.

[20] L. Vandezande, L. Meeus, R. Belmans, M. Saguan, and J. M. Glachant. Well-functioning balancing markets: A prerequisite for wind power integration. *Energy Policy*, 38(7):3146–3154, 2010.

[21] J.M. Morales, A.J. Conejo, and J. Perez-Ruiz. Short-Term Trading for a Wind Power Producer. *Power Systems, IEEE Transactions on*, 25(1):554–564, feb. 2010.

[22] M. Carrión, A.J. Conejo, and J.M. Arroyo. Forward Contracting and Selling Price Determination for a Retailer. *Power Systems, IEEE Transactions on*, 22(4):2105–2114, 2007.

[23] M. Rahimiyan, J. M. Morales, and A. J. Conejo. Evaluating alternative offering strategies for wind producers in a pool. *Applied Energy*, 88(12):4918–4926, 2011.

[24] J. M. Morales, A. J. Conejo, K. Liu, and J. Zhong. Pricing Electricity in Pools With Wind Producers. *Power Systems, IEEE Transactions on*, PP(99):1–11, 2012.

[25] J. P. S. Catalão, H. M. I. Pousinho, and V. M. F. Mendes. Optimal Offering Strategies for Wind Power Producers Considering Uncertainty and Risk. *Systems Journal, IEEE*, 6(2):270–277, June 2012.

[26] R.A. Jabr. Robust self-scheduling under price uncertainty using conditional value-at-risk. *Power Systems, IEEE Transactions on*, 20(4):1852–1858, 2005.

[27] A. Tuohy, P. Meibom, E. Denny, and M. O'Malley. Unit Commitment for Systems With Significant Wind Penetration. *Power Systems, IEEE Transactions on*, 24(2):592–601, 2009.

[28] Directive 2009/28/EC of the European Parliament and of the Council of 23 April 2009 on the promotion of the use of energy from renewable sources and amending and subsequently repealing Directives 2001/77/EC and 2003/30/EC.

[29] E.Y. Bitar, R. Rajagopal, P.P. Khargonekar, K. Poolla, and P. Varaiya. Bringing Wind Energy to Market. *Power Systems, IEEE Transactions on*, 27(3):1225–1235, 2012.

[30] J. Rivier. Electricity market participation of wind farms: the success story of the Spanish pragmatism. *Energy Policy*, 38(7):3174–3179, 2010.

[31] Y.G. Rebours, D.S. Kirschen, M. Trotignon, and S. Rossignol. A Survey of Frequency and Voltage Control Ancillary Services – Part I: Technical Features. *Power Systems, IEEE Transactions on*, 22(1):350–357, 2007.

[32] UCTE - Union for the Coordination of the Transmission of Electricity (2009, Marth, 19th). Operation Handbook - Policy 1: Load-Frequency Control and Performance (V3.0 rev 15) [Online]. Available: https://www.entsoe.eu.

[33] S. Spiecker and C. Weber. Integration of fluctuating renewable energy; A German case study. In *Power and Energy Society General Meeting, 2011 IEEE*, July 2011.

[34] S. Bofinger, A. Luig, and H. G. Beyer. Qualification of wind power forecasts. In *Global Wind Power Conference*, Paris, France, Apr. 2-5 2002.

[35] A. Fabbri, T.G.S. Román, J.R. Abbad, and V.H.M. Quezada. Assessment of the Cost Associated With Wind Generation Prediction Errors in a Liberalized Electricity Market. *Power Systems, IEEE Transactions on*, 20(3):1440–1446, 2005.

[36] M.A. Ortega-Vazquez and D.S. Kirschen. Estimating the Spinning Reserve Requirements in Systems With Significant Wind Power Generation Penetration. *Power Systems, IEEE Transactions on*, 24(1):114–124, 2009.

[37] R.T. Rockafellar and S. Uryasev. Optimization of conditional value-at-risk. *Journal of Risk*, (2):21–41, 2000.

[38] GAMS Development Corporation. GAMS Base Module 23.9.4 (LEX 35892.35906 LEG x86_64/Linux). 1217 Potomac Street, NW, Washington, DC 20007, USA, 4 July 2012.

[39] Spanish Ministry of Industry, Energy and Tourism. Register of generators in the Spanish power system, Consulted on September 2013. [Online]. Available: http://www.minetur.gob.es/.

[40] J. Bertsch, C. Growitsch, S. Lorenczik, and S. Nagl. Flexibility in Europe's power sector -an additional requirement or an automatic complement? EWI Working Paper No 13/10, Institute of Energy Economics at the University of Cologne (EWI), June 2013.

[41] M. G. Lijesen. The real-time price elasticity of electricity. *Energy Economics*, 29(2):249 – 258, 2007.

[42] J.C. Smith, M.R. Milligan, E.A. DeMeo, and B. Parsons. Utility Wind Integration and Operating Impact State of the Art. *Power Systems, IEEE Transactions on*, 22(3):900–908, 2007.

[43] M. Milligan, P. Donohoo, D. Lew, E. Ela, B. Kirby, H. Holttinen, E. Lannoye, D. Flynn, M. O'Malley, N. Miller, P. B. Eriksen, A. Gottig, B. Rawn, M. Gibescu, E. Gómez, A. Robitaille, and I. Kamwa. Operating Reserve and Wind Power Integration: An International Comparison. In *The 9th Annual International Workshop on Large-Scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants Conference, Québec, Canada*, number NREL/CP-5500-49019. National Renewable Energy Laboratory (NREL), 18-19 October 2010.

[44] P. Meibom, R. Barth, B. Hasche, H. Brand, C. Weber, and M. O'Malley. Stochastic Optimization Model to Study the Operational Impacts of High Wind Penetrations in Ireland. *Power Systems, IEEE Transactions on*, 26(3):1367–1379, 2011.

[45] Agora Energiewende. 12 Insights on Germany's Energiewende. Discussion paper 010/03-1-2013/EN, Agora Energiewende, Rosenstrasse 2, 10178 Berlin, Germany, February 2013.

[46] M. Zugno, J.M. Morales, P. Pinson, and H. Madsen. Pool strategy of a price-maker wind power producer. *Power Systems, IEEE Transactions on*, 28(3):3440–3450, Aug 2013.

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